PVP 23

Code: 23BS1101

I B.Tech - I Semester – Regular Examinations - JANUARY 2024

LINEAR ALGEBRA & CALCULUS
(Common for ALL BRANCHES)

Duration: 3 hours Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

BL – Blooms Level CO – Course Outcome

PART – A

		BL	CO
1.a)	Estimate the value of a , if the rank of the matrix $A=\begin{bmatrix}1 & 2 & 3\\0 & a & 4\\1 & -1 & 1\end{bmatrix}$ is 2	L2	CO1
1.b)	If the initial approximation to the solution of $10x+2y+z=9,\ 2x+20y-2z=-44,\ -2x+3y+10z=22$ is $(x,y,z)=(0,0,0)$ then find the first approximation by using Gauss-Seidel iteration method.	L3	C04
1.c)	If the eigen values of $A=\begin{bmatrix}3 & -1 & 1\\-1 & 5 & -1\\1 & -1 & 3\end{bmatrix}$ are 2, 3 & 6 then predict the eigen values of A^{-1}.	L2	CO2
1.d)	Write down the quadratic form $X^T AX$ corresponding to the symmetric matrix $A=\begin{bmatrix}1 & 3 & -5\\3 & 2 & 0\\-5 & 0 & -4\end{bmatrix}$	L2	CO4
1.e)	Discuss the applicability of Cauchy's mean value theorem for $f(x)=\begin{cases}-x\ , & if -4 < x < 0\\ x\ , & if\ \ 0 \le x < 4\end{cases}$ and $g(x)=x^2$ in $[-4,4]$	L2	CO3

1.f)	State the Maclaurin's series expansion of $f(x)$ about $x=0$.	L1	CO3
1.g)	Estimate $\lim\limits_{\substack{x\to 1\\ y\to 2}} \dfrac{2x^2y}{x^2+y^2+1}$	L2	CO1
1.h)	Estimate the first and second order partial derivatives of $f(x,y)=ax^2+2hxy+by^2$	L2	CO1
1.i)	Write the limits by changing the order of integration of the double integral $\int_0^1\int_y^{y^2}(x+y)\ dxdy$ with the help of region of integration.	L2	CO5
1.j)	Calculate the double integral $\int_0^1\int_0^1 xy\ dydx$.	L3	CO5

PART – B

			BL	CO	Max. Marks
		UNIT-I			
2	a)	Discover the rank of the matrix $\begin{bmatrix} 1 & 1 & -1 & 1 \\ -1 & 1 & -3 & -3 \\ 1 & 0 & 1 & 2 \\ 1 & -1 & 3 & 3 \end{bmatrix}$ by reducing the matrix to Echelon form.	L3	CO2	5 M
	b)	Solve the system of non-homogeneous linear equations $5x_1+3x_2+7x_3=4$, $3x_1+26x_2+2x_3=9$ and $7x_1+2x_2+10x_3=5$	L3	CO2	5 M
		OR			
3	a)	Apply Gauss Jordan method to find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$	L3	CO2	5 M
	b)	Make use of Jacobi's method to find first five iterations of the following system of equations $20x+y-2z=17$, $3x+20y-z=-18$, $2x-3y+20z=25$	L3	CO2	5 M

UNIT-II					
4	a)	Calculate the characteristic roots and characteristic vectors of the matrix $A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$	L3	CO2	5 M
	b)	Make use of the eigen values of matrix of the quadratic form to discuss the rank and nature of the quadratic form $-x_1^2-4x_2^2-x_3^2+4x_1x_2-4x_2x_3-2x_1x_3$	L4	CO4	5 M
OR					
5	a)	Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ and hence find A^4.	L3	CO2	5 M
	b)	Use Diagonalization to find the matrix *A*, if the eigen values of a matrix A of order 3 and the corresponding eigen vectors are 0,3,15 & $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ respectively.	L3	CO2	5 M
UNIT-III					
6	a)	Check the applicability of Rolle's theorem, if applicable verify theorem for the function $\log\left\{\frac{x^2+ab}{x(a+b)}\right\}$ in $[a,b]$, where $0<a<b$	L3	CO5	5 M
	b)	Construct the series expansion of $f(x)=\log(1+x)$ in powers of x up to third degree terms.	L3	CO5	5 M
OR					
7	a)	Apply mean value theorem to prove that $\frac{b-a}{1+b^2} < tan^{-1} b - tan^{-1} a < \frac{b-a}{1+a^2}$ $(0<a<b)$ and hence deduce that $\frac{\pi}{4}+\frac{3}{25} < tan^{-1}\frac{4}{3} < \frac{\pi}{4}+\frac{1}{6}$.	L3	CO5	5 M

	b)	Discover the series expansion of $f(x) = Sin\,x$ in powers of $x - \frac{\pi}{4}$	L3	CO5	5 M
UNIT-IV					
8	a)	Point out the functions $u = x\,e^y sin\,z,\ \ v = x\,e^y cos\,z,\ w = x^2 e^{2y}$ are functionally dependent or not. If functionally dependent, find the relation between them.	L3	CO5	5 M
	b)	Discover the nature of stationary points and then find extreme values of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$	L3	CO3	5 M
OR					
9	a)	Make use of functional determinant to show that $\frac{\partial(u,v)}{\partial(r,\theta)} = 6r^3 \sin 2\theta$ where $u = x^2 - 2y^2, v = 2x^2 - y^2$ and $x = r\cos\theta, y = r\sin\theta$	L3	CO5	5 M
	b)	Divide twenty-four into three parts such that the continued product of the first part, square of the second part and the cube of third part is maximum.	L4	CO3	5 M
UNIT-V					
10	a)	By changing the order of integration, evaluate the double integral $\int_0^2 \int_{e^x}^{e} \frac{1}{log\,y}\,dy\,dx$	L3	CO5	5 M
	b)	Calculate the volume of the solid bounded by the planes $x\ = 0, y = 0, z = 0\ and\ x + y + z = 1.$	L3	CO3	5 M
OR					
11	a)	Calculate the triple integral $\int_{-1}^{1} \int_0^2 \int_1^3 x^2 y^2 z^3\,dx\,dy\,dz.$	L3	CO5	5 M
	b)	Discover the area enclosed by the pair of curves $y^2 = x$ and $y = x^2$ using double integration.	L3	CO3	5 M